Emerging Markets Finance and Trade

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/mree20

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To cite this article: Gabriel Augusto de Carvalho, Hudson Fernandes Amaral, Juliano Lima Pinheiro & Laíse Ferraz Correia (2021): Pricing of Liquidity Risk: New Evidence from the Latin American Emerging Stock Markets, Emerging Markets Finance and Trade, DOI: 10.1080/1540496X.2021.1991184

To link to this article: https://doi.org/10.1080/1540496X.2021.1991184

Published online: 06 Nov 2021.

EMERGING MARKETS FINANCE AND TRADE
https://doi.org/10.1080/1540496X.2021.1991184

Pricing of Liquidity Risk: New Evidence from the Latin American Emerging Stock Markets

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ABSTRACT
This paper aims to analyze whether the liquidity risk is priced in Latin-American emerging stock markets. For that, we test the performance of the liquidity augmented version of Fama-French three and five factor models and Carhart four factor model since there is not yet a consensus about their suitability for these markets. Two versions of a liquidity factor were constructed based on two proxies that consider different dimensions of liquidity and are more appropriate for low frequency data. The GRS statistics showed Latin American average returns are better explained by the liquidity augmented Fama-French five-factor model. When estimated by GMM-IV$_{d,}$ due to the possible endogenous problems caused by liquidity, the results of the models did not significantly change. The results were robust to the January Effect. Furthermore, when the sample period was divided into two subperiods, both were statistically significant, although the explanatory power was greater in the second subperiod.

KEYWORDS
Stock liquidity; asset pricing; Fama-French five-factor model; emerging markets

JEL
G11; G12; G15

1. Introduction

Asset pricing literature has shown abundant evidence of the multidimensional nature of risk (e.g. Roll and Ross (1980), Chen, Roll, and Ross (1986), Amihud and Mendelson (1986), Fama and French (1993, 2015) and Carhart (1997)). Along these lines, stock returns would be explained by several components, instead of a single factor as in Sharpe (1964), Lintner (1965) and Mossin (1966).

In efficient markets, stock returns should be unpredictable and financial assets should generate only risk-adjusted returns. Nonetheless, as Atilgan, Demirtas, and Simsek (2015) and Atilgan, Demirtas, and Gunaydin (2020) argue, finance empirical research has documented that some firm-characteristics are cross-sectionally correlated with expected returns in the United States, which are called market anomalies. The effects of the risks generated by stock characteristics such as size, book-to-market (value), momentum, investment and profitability are examples of explanatory factors incorporated into the most recent asset pricing models: Fama and French (1993, 2015) and Carhart (1997).

Inspired by the empirical evidences, several studies were carried out with the aim to test if one or more anomalies could predict stock returns. For instance, Narayan and Bannigidadmath (2015), Bannigidadmath and Narayan (2016), and Narayan, Phan, and Bannigidadmath (2017) test the stock return predictability in the Indian market. Kim, Kim, and Park (2020) studied the effect of various anomaly variables in the Korean market. Atilgan, Demirtas, and Gunaydin (2020) analyze

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whether various firm-specific attributes have predictive power on future stock returns for a sample of stocks from 23 emerging markets (characteristic-based model). In general, these papers find some evidence of return predictability associated with various anomaly variables using data from emerging market.

Among the various anomalies assessed in asset pricing literature, we highlight the market liquidity effect, defined by Amihud and Mendelson (1986) as the ease of trading an asset without major losses and costs. These researchers were among the first to propose that low-liquidity stocks be traded at a discount relative to high-liquidity stocks. They argue that such stocks would thus become more attractive to investors, who would bear the cost of illiquidity to obtain higher returns. Considering that the liquidity risk cannot be eliminated through the diversification of portfolios, a rational investor should, therefore, balance the costs of illiquidity according to their investment time horizon.

Liquidity is characterized as a multidimensional variable, which is measured using different proxies, such as the bid-ask spread, the turnover ratio and the measurements proposed by Amihud (2002), Liu (2006) and Kang and Zhang (2014). This is because, as Amihud and Mendelson (2008) point out, the liquidity cost of an asset has three components: (i) direct transaction costs; (ii) market impact costs (indirect costs of the transaction, which reflect the price concession made by the parties of a transaction to execute it, such as bid-ask spread); and (iii) search and delay costs.

Given the importance of market liquidity for the investors' decision-making, several studies, such as Amihud and Mendelson (1986, 1988), Brennan and Subrahmanyam (1996), Amihud (2002), Datar, Naik, and Radcliffe (1998), Acharya and Pedersen (2005) and Liu (2006) have analyzed its effects on asset pricing in the North American stock market. In general, empirical evidence suggests an inverse relationship between market liquidity and stock returns.

Research on the liquidity premium as part of asset pricing models has been conducted, mostly, with data from the US market (Amihud et al. 2015). We are among the authors who argue that, if not taken into account, the peculiarities of emerging markets can compromise the asset pricing. Namely, asset pricing models would be unable to satisfactorily explain the cross-section of average returns on stocks in these markets. As Leite et al. (2018) state, emerging markets are characterized by the lower quality of available data, political and institutional instability, and greater vulnerability to speculative capital. These particularities challenge the assessment of the factors that influence asset prices. For instance, Harvey (1995) considers that assets traded in emerging markets have a small degree of exposure to the pricing factors traditionally used in the literature. Hence, rationality in investors' decision making and the capacity of the pricing models to describe the return on assets in these markets have specificities inherent to them.

Cakici, Fabozzi, and Tan (2013), Cakici, Tang, and Yan (2016), Zaremba and Czapkiewicz (2017), Foye (2018), Leite et al. (2018) and Altay and Çalgıcı (2019) are examples of recent studies that have relied on data from emerging stock markets to analyze asset pricing empirically. In general, their results point out to a lower power of asset pricing models in explaining cross-sectional patterns (anomalies) in emerging markets, in addition to presenting evidence of the segmentation of such markets when compared to developed economies.

The empirical evidences are controverse when considering the liquidity effect on emerging markets stock returns. Unlike what has been proposed by Amihud and Mendelson (1986), Jun, Marathe, and Shawky (2003) have documented a positive and significant relationship between liquidity and stock returns. Nonetheless, Amihud et al. (2015) found a negative relationship between liquidity and stock returns and also found that the illiquidity premium in emerging markets is higher.

Thus, the relevance of testing the liquidity effect in asset pricing models in the context of emerging markets is highlighted. The main purpose of this research was to test the performance of Fama-French (1993, 2015) and Carhart (1997) liquidity augmented asset pricing models using data from a wide sample of stocks traded on the Latin American emerging markets. As a secondary objective, this research sought to test whether the systematic liquidity risk is a priced factor in the cross-section stock returns in these markets.

We are also interested in the suitability of Fama-French asset pricing models for the emerging markets since this is still an open question. Thus, the three and five Fama-French and the Carhart factor models were used here. Hence, it was possible not only to evaluate, but also to compare the effect of including a liquidity factor among other risk factors. In doing so, we provided a better understanding of the cross-section returns patterns in emerging markets. Other models such the one applied in Narayan and Zheng (2011) could have been used. Nevertheless, this would rule out comparison with other evidence on the asset pricing in Latin-American stock markets as the ones from Cakici, Fabozzi, and Tan (2013), Foye (2018) and Leite et al. (2018).

We contribute to the literature in two main ways: (i) by addressing jointly the liquidity premium and the asset pricing issues in the Latin American emerging markets, notably with the liquidity augmented Fama-French five factor model, that is still scarcely studied in these markets; and, (ii) by taking into account two different liquidity proxies developed specially to address the phenomena in low-liquidity markets, as the Kang and Zhang (2014) measure.

Our results showed that stock liquidity is a priced risk factor in such markets, even after other relevant factors of previous studies were considered. When estimated by GMM-IV$_d$, due to the possible endogenous problems caused by liquidity, the results of the models did not significantly change. The estimates were robust to the January Effect. When the models were estimated with the total period divided into two subperiods, greater statistical significance for the coefficients in the second one was found. Therefore, the results of this study are expected to contribute to the decision making of financial market players, portfolio managers and capital market regulators, so that each of these agents may consider stock liquidity aspects and, consequently, minimize the costs resulting from illiquidity.

This article is organized into five sections including this introduction. Section 2 presents a literature review on liquidity risk and its inclusion in asset pricing models. Section 3 describes the research sample and the methodological procedures adopted for estimating and testing the models. Section 4 discusses the empirical results. Lastly, Section 5 presents the final remarks on the results as well as their theoretical and practical implications.

2. Literature Review

The pioneering studies investigating the effect of market liquidity on asset pricing were based on the theoretical framework of the CAPM developed by Sharpe (1964), Lintner (1965) and Mossin (1966). For such, they generally adopted Demsetz's (1968) bid-ask spread as an illiquidity measure. For Amihud and Mendelson (1986, 1989) have used the bid-ask spread to assess whether (and how) liquidity affects stock returns. As a result, the authors observed a concave and increasing relationship between stock returns and illiquidity. Subsequent research employed various pricing models and proxies for stock liquidity and generally found an inverse relationship between liquidity and stock returns.

Due to the multidimensional character of liquidity, as pointed out by Amihud and Mendelson (2008), different proxies have been proposed to measure this variable. Among them, (i) Datar, Naik, and Radcliffe (1998) used the turnover ratio to measure liquidity; (ii) Amihud (2002) proposed a new proxy to measure illiquidity, based on the ratio between the absolute return and the daily trading volume of a given asset; and (iii) Liu (2006) proposed a turnover ratio adjusted by the number of non-trading days of a given asset. In addition, as pointed out by Amihud (2019), liquidity measures may behave differently according to market conditions, such as in crisis scenarios, which can affect analyzes depending on the proxy employed.

With the development of empirical tests indicating the relevance of the liquidity risk effect, studies came to incorporate a liquidity factor in the different asset pricing models. For instance, Acharya and Pedersen (2005) proposed a liquidity-adjusted CAPM, by adding beta coefficients representing liquidity risk. The authors' results showed that the new model had greater explanatory power compared to the traditional CAPM. Following the same approach, Liu (2006) tested a model consisting of the beta factor of the CAPM and the liquidity risk factor. The results showed that the inclusion of a stock liquidity factor was able to increase the explanatory power of both the CAPM and the Fama-French three-factor model (1993).

Based on the Fama-French three-factor model (1993) and the Carhart model (1997), Keene and Peterson (2007) analyzed the role of stock liquidity in explaining the portfolio's excess return, both in isolation and incorporating the effects of the factors market, size, value, and momentum. The test statistics showed the relevance of stock liquidity: the liquidity factor remained significant even after considering the effect of the other pricing factors.

The aforementioned empirical tests relied on data from the North American stock market. However, the effect of stock liquidity on asset pricing models was also assessed using data from other countries, both developed and emerging. For instance, Bekaert, Harvey, and Lundblad (2007) showed that liquidity risk is a priced factor in a sample of 18 emerging markets, thus proving to be more relevant than the market risk. Narayan and Zheng (2010) demonstrated that liquidity is a significant variable in asset pricing for a sample of stocks listed on the Shanghai Stock Exchange. Machado and Medeiros (2011) showed the existence of a liquidity premium for stocks listed on the Brazilian Stock Exchange. They also concluded that the incorporation of a liquidity risk factor into the Carhart model (1997) was able to increase its explanatory power. Lam and Tam (2011) showed that liquidity is a priced factor for a sample of stocks listed on the Hong Kong Stock Exchange. Besides, they also observed the existence of higher premiums for the liquidity factor compared to the values identified by Keene and Peterson (2007) when studying the North American stock market.

In more recent studies, Amihud et al. (2015) analyzed the effect of liquidity on the returns of stocks listed on the stock exchanges of 45 countries. They found a positive and significant value for the illiquidity premium, which was higher in the stock markets of emerging countries. Vu, Chai, and Do (2015) adopted the model proposed by Acharya and Pedersen (2005) and demonstrated that liquidity is priced in the Australian stock market. They also found a negative relationship between liquidity and stock returns.

In disagreement with the results prevailing in the literature, there is both empirical evidence of a positive relationship between liquidity and returns, and of no association between these variables. For instance, Jun, Marathe, and Shawky (2003) analyzed data from 27 emerging markets and found a positive relationship between liquidity and stock returns. This direct relationship was attributed to the lower level of integration of emerging markets into the global economy. In turn, Lischewski and Voronkova (2012) showed that a liquidity factor was not able to increase the explanatory power of the models tested for a sample of stocks listed on the Warsaw Stock Exchange.

Given the heterogeneity of evidence regarding the effect of stock liquidity on asset pricing models tested on emerging market, this study analyzes the impact of adding a liquidity risk factor to the Fama-French five-factor model (2015), as well as the models by Fama and French (1993) and Carhart (1997), in the context of Latin American emerging markets.

3. Methodology

3.1. Sample and Data

The sample of this study comprises the stock markets of Brazil, Chile, Colombia, Mexico and Peru, which are the countries included in the *Morgan Stanley Capital International* (MSCI) *Emerging Markets Latin America Index* dated August 2018. The necessary data to construct the factors and estimate the models were collected from Bloomberg, and fully converted to US dollars. To estimate the models, monthly stock returns of portfolios between July 2000 and June 2018 were used.

In the final sample of this research were considered only stocks of non-financial firms. As pointed out by Fama and French (1992), the high leverage of these companies can affect the interpretation of the indices analyzed. In addition, were considered only those stocks that had data for book equity and operating profit in December of the previous year; data for book assets in December of the previous two years; and stocks with at least one transaction for each month, corresponding to the 12 months before and after the date of the portfolios' formation.

Table 1 shows the annual composition of the sample. In all years, the largest number of assets in the sample comes from the Brazilian market, and the smallest, from the Colombian market. We highlight that the number of assets eligible for the sample grew at the beginning of the period studied until reaching its peak in 2011, with the total sample being composed of 438 assets. After that, there was a reduction in the number of assets in the sample, and this variation was largely influenced by Brazil, which went through a period of economic and political crisis, which negatively affected its capital market.

3.2. Liquidity Proxies

We used two stock liquidity proxies to obtain the liquidity risk factor to be included in the models considered in this study.

Adjusted Illiquidity ($\mathrm{AdjIlliq}_{i,t}$), proposed by Kang and Zhang (2014). The $\mathrm{AdjIlliq}_{i,t}$, can be expressed by Equation (1),

$$\mathrm{AdjIlliq}_{i,t} = \left[\ln\left(\frac{1}{N_{i,t}}\sum_{d=1}^{N_{d,t}}\frac{|R_{i,d}|}{\mathrm{Vol}_{i,d}}\right)\right] \times \left(1 + \mathrm{ZeroVol}_{i,t}\right) \tag{1}$$

where $\mathrm{AdjIlliq}_{i,t}$ is the absolute return of the stock *i* in the year *t* over the trading volume in the same period, adjusted by the non-trading days of the stock *i*; $N_{i,t}$ is the number of non-trading days of the stock *i* in the year *t*; $|R_{i,d}|$ is the absolute return of the stock *i* on the day *d*; $\mathrm{Vol}_{i,d}$ is the trading volume of the stock *i* on the day *d*; and $\mathrm{ZeroVol}_{i,t}$ is the percentage of non-trading days for the stock *i* in the year *t*.

Standardized Turnover Ratio (ST), proposed by Liu (2006). In this study, this variable has been adjusted according to the non-trading days of the stock *i* during the previous 12 months. The ST Ratio can be mathematically described by Equation (2),

$$\mathrm{ST}_{i,t} = \left[X + \frac{\frac{1}{Z}}{11,000}\right] \times \frac{21 \times 12}{Y} \tag{2}$$

Table 1. Annual sample composition.

	Brazil	Chile	Colombia	Mexico	Peru	Total
2000	109	70	8	65	21	273
2001	119	63	8	69	25	284
2002	123	61	9	61	29	283
2003	128	65	9	58	31	291
2004	141	64	9	58	35	307
2005	144	68	7	60	40	319
2006	161	78	6	60	47	352
2007	178	88	6	58	48	378
2008	211	81	5	58	51	406
2009	229	76	7	66	49	427
2010	231	84	7	67	47	436
2011	223	92	8	71	44	438
2012	233	84	9	69	37	432
2013	224	84	12	67	35	422
2014	215	86	19	66	31	417
2015	201	82	17	70	26	396
2016	192	78	20	73	27	390
2017	197	80	21	71	28	397

where, $ST_{i,t}$ is the Standardized Turnover Ratio for the stock *i* in the year *t*; X is the number of non-trading days of the stock *i* in the previous 12 months; Y is the number of trading days in the market in the same period; Z is the turnover ratio in the previous 12 months, calculated as the sum of the daily turnover ratio for the period. This, in turn, is given by the ratio between the number of stocks traded on a given day and the number of stocks outstanding at the end of the same day.

Liu (2006) suggests the adoption of a deflator of 11,000 to calculate the turnover ratio for 12 months. This value is adopted so that $0 < \frac{\frac{1}{Z}}{11{,}000} < 1$ for every stock in the sample. The term $\frac{21 \times 12}{Y}$ is used to standardize the number of trading days in the month at 21, which makes this liquidity measure comparable over time.

3.3. Econometric Models

The econometric models estimated in this research are described by Equations (3)–(5). All models were estimated twice since we used two liquidity measures, and for this reason two alternative liquidity risk factors were constructed.

Fama-French three-factor model (1993):

$$R_{i,t} - RF_t = \alpha_i + \beta_i(RM_t - RF_t) + s_i SMB_t + h_i HML_t + l_i IML_t + \varepsilon_{i,t} \tag{3}$$

Carhart four-factor model (1997):

$$R_{i,t} - RF_t = \alpha_i + \beta_i(RM_t - RF_t) + s_i SMB_t + h_i HML_t + w_i WML_t + l_i IML_t + \varepsilon_{i,t} \tag{4}$$

Fama-French five-factor model (2015):

$$\begin{aligned} R_{i,t} - RF_t = {} & \alpha_i + \beta_i(RM_t - RF_t) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \\ & l_i IML_t + \varepsilon_{i,t} \end{aligned} \tag{5}$$

where, $R_{i,t}$ is the return of portfolio *i* in the month *t*; RF_t is the return on the risk-free asset, supposed as the return of one-month U.S. Treasury Bill; RM_t is the return on the market portfolio, represented by value-weighted return of a portfolio composed of all stocks in the sample in each year of the sample period; SMB_t is the return difference between portfolios consisting of small and large market capitalization stocks; HML_t is the return difference between portfolios consisting of high and low book-to-market stocks; WML_t is the return difference between portfolios consisting of winners and losers stocks; RMW_t is the return difference between the portfolios consisting of firms with high and low operating profitability; CMA_t is the return difference between portfolios consisting of low and high investment firms; IML_t is the return difference between portfolios consisting of low and high liquidity stocks.

The intercept coefficients of the models were obtained from ordinary least squares time-series models. Based on these estimates, we tested whether the addition of a liquidity risk factor can increase the explanatory power of the models. As highlighted by Fama and French (2015), if the factors of a model can fully explain the expected return, the intercept must be statistically equal to zero. For this purpose, the GRS statistic by Gibbons, Ross, and Shanken (1989) was used to jointly test whether all estimated intercepts are statistically equal to zero (null hypothesis). The rejection of this hypothesis implies that there are other sources of risk not captured by the factors of the estimated model.

As a robustness test, all of the asset pricing models tested in this paper were estimated again with the sample divided into two equal subperiods of 108 months. And, to verify whether the liquidity effect is restricted to the month of January, as observed by Eleswarapu and Reinganum (1993), the estimates of the models were obtained without considering the data for that month.

Since liquidity factors used in asset pricing models are not observed but constructed from proxies, they are not free from measurement errors. As pointed out by Adrian et al. (2017), liquidity is an endogenous variable. Thus, it is necessary to adopt appropriate methodologies to deal with this type of problem in econometric models. To take the liquidly endogeneity problem into account, we also estimate the models using the generalized method of moments robust instruments (GMM-IV$_d$) proposed by Racicot (2015) and implemented in asset pricing models with liquidity by Racicot and Rentz (2015) and Racicot et al. (2019).

To check the validity of the instruments used in this robustness test, we performed the Olea and Pflueger (2013) weak instrument test. The null hypothesis of weak instruments, considering a critical value of 5% (tau), could not be rejected only for the instruments used in the Fama-French three-factor model with liquidity proxied by Standardized Turnover Ratio. Thus, we also present the estimates for the GMM-IV$_d$ in this paper (Appendix B).

3.4. Independent Variables

To construct the models' Right-Hand-Side (RHS) factors – independent variables –, a procedure similar to that of Fama and French (1993, 2015) was adopted. Firstly, for each year *t*, the stocks were sorted in ascending order by their market value in June of the year *t* and then divided into two groups, based on the median value. Secondly, the stocks were independently ranked again (second ranking), according to the values of the other characteristics of the pricing models adopted herein, along with the stock's liquidity. Finally, each of the groups was subdivided into three other ones based on the 30th and 70th percentiles.

This procedure resulted in six portfolios for each characteristic used in the second ranking (book-to -market ratio, momentum, profitability, investment, Adjusted Illiquidity and Standardized Turnover Ratio), as detailed in Table 2. It should be noted that the SMB (*Small Minus Big*) factor used to estimate the three and four-factor models consisted only of $SMB_{B/M}$, as in the works of Fama and French (1993) and Carhart (1997).

The portfolio formation procedure was performed at the end of each June between 2000 and 2017 so that the latest information could be incorporated into their construction. Therefore, the monthly value-weighted returns on the portfolios were calculated from July of the year *t* to June of the year $t + 1$.

Besides the aforementioned factors, the models also incorporate the market risk premium, obtained by calculating the difference between the monthly value-weighted return of the market portfolio (RM) and the monthly risk-free interest rate (RF). RM was composed of all stocks in the sample in each year of the sample period. RF is the return of one-month U.S. Treasury Bill.

3.5. Dependent Variables

To construct the *Left-Hand-Side* (LHS) portfolios, a procedure similar to that of Fama and French (2015) was adopted. By double ranking the stocks, five sets of portfolios were formed to estimate the models. Initially, the stocks were ranked by their size and then divided into quintiles. After that,

Table 2. RHS factors construction procedure.

Variable	Classification	Factor
Size	Median	$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN + BL)/3$ $SMB_{Luc} = (SR + SN + SW)/3 - (BR + BN + BW)/3$ $SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3$ $SMB = \left(SMB_{B/M} + SMB_{Luc} + SMB_{Inv}\right)/3$
Book-to-Market	30th and 70th percentiles	
Momentum	30th and 70th percentiles	$WML = (SWin + BWin)/2 - (SLos + BLos)/2$
Profitability	30th and 70th percentiles	$RMW = (SR + BR)/2 - (SW + BW)/2$
Investment	30th and 70th percentiles	$CMA = (SC + BC)/2 - (SA + BA)/2$
Liquidity	30th and 70th percentiles	$IML = (SI + BI)/2 - (SL + BL)/2$

The portfolios were formed by carrying out a double ranking procedure, ordering the stocks, and dividing them into 2×3 groups. The first ranking was always oriented by Size, and the second one by the other characteristics analyzed in this paper. After the second ranking, the factors were constructed from the return difference between portfolios with high and low characteristic (30th and 70th percentiles) of the second ranking.

each quintile was ranked by a second variable: (i) book-to-market ratio; (ii) profitability; (iii) investment; (iv) Adjusted Illiquidity; and (v) Standardized Turnover. Finally, these quintiles were divided again into other quintiles. This procedure resulted in 125 portfolios. The number of stocks in these portfolios ranged from a minimum of 10 to a maximum of 19, with a mean of 14.77 stocks. The dependent variables of the models were represented by the excess returns of these portfolios over the risk-free interest rate, where the returns of the portfolios were weighted by the market value of their stocks.

4. Regression Inputs

4.1. LHS Portfolio Returns

An initial step in asset pricing analysis involves the formation of portfolios to calculate their returns that will make up the dependent variable of the models. In addition, the analysis of the portfolio's return behavior allows us to infer about the relationship between the variables used in the rankings performed in the process of their formation. Table 3 shows the mean and standard deviation of the excess return of the 125 LHS portfolios. Each panel shows the values for a different group of LHS portfolios, organized according to the characteristic adopted as of the second ranking during the portfolio formation procedure.

In relation to the presence of the size effect, when analyzing the columns of the matrix for the mean in all Panels in Table 3, a downward trend in excess return was observed as portfolios composed of stocks with higher market capitalization are considered. A similar pattern was also observed by Fama and French (2015) in developed markets and, in the context of emerging markets in Latin America, by Cakici, Fabozzi, and Tan (2013) and Leite et al. (2018).

The relationship between return and book-to-market ratio can be observed in each row of Panel (a) of Table 3. Similar to the results of Fama and French (2015) and Cakici, Fabozzi, and Tan (2013), we noticed an increasing trend in the average returns from the *Low* to the *High* book-to-market portfolios. The book-to-market ratio effect is more significant in the portfolios with the lowest market capitalization (*Small*), when compared to those with the highest market capitalization (*Big*). In the *Small* group, the average return increased from 0.8436% to 2.2637% from the *Low* to the *High* book-to-market quintiles, while in the *Big* group it increased from 0.4209% to 0.7213%.

As for the investment effect there is a different pattern from that of Fama and French (2015). As you can see in Panel (b) the average return of the *Low* portfolios was higher than that of the *High* portfolios only for the *Small*, 2 and 3 quintiles, according to size. This result indicates that the investment effect is not present in the groups of stocks formed by market capitalization in the analyzed sample.

In Panel (c) we can see the relationship between return and profitability, when comparing the extremes (*Low* and *High*), the *High* group always presented higher returns, corroborating the results of Fama and French (2015). It should be noted that the profitability effect was lower for the *Big* group portfolios, reaching a mean of 0.3354% in the *Low* portfolio and 0.5517% in the *High* portfolio. This relationship differs from that found by Leite et al. (2018) for Latin America emerging stock markets, as these authors observed higher average returns for low-profit portfolios (*Low*).

Panels (d) and (e) present the average and standard deviation of the excess return of portfolios formed according to Size and Liquidity (AdjIlliq and ST, respectively). As AdjIlliq and ST are illiquidity measures, their rankings have been inverted to reflect stock liquidity, so that the *High* quintile would consist of more liquid stocks and the *Low* of less liquid ones. In turn, by analyzing the relationship between liquidity and excess return, we find, as expected, that the portfolios with the least liquid stocks (*Low*) had higher returns than those in the most liquid portfolios (*High*). The only exception was in the *Big* portfolios when considering ST. In this case, the average return of the portfolios with more liquid stocks was higher than the portfolio with less liquid stocks – 0.6408% (*High*) and 0.6254% (*Low*), respectively.

Table 3. Average and standard deviation of the monthly returns of the LHS portfolios.

Panel (a): Portfolios formed by size/book-to-market ratio

	Mean					Standard deviation				
	Low	2	3	4	High	Low	2	3	4	High
Small	0.8436	1.6775	1.8581	2.0105	2.2637	9.0437	8.3484	9.6646	9.0696	8.4727
2	1.2731	1.2952	1.7941	2.7619	1.8136	8.8339	7.4174	7.8206	8.5000	9.3274
3	0.5159	1.3284	0.9358	1.5069	1.5458	8.1598	7.1883	8.0903	7.8140	8.6186
4	0.9368	0.8417	1.1315	1.0921	1.1476	6.8290	6.6835	6.6623	7.3880	8.3322
Big	0.4209	0.3880	0.4499	0.9325	0.7213	7.1090	6.6723	7.0559	7.9369	9.6569

Panel (b): Portfolios formed by size/investment

	Mean					Standard deviation				
	Low	2	3	4	High	Low	2	3	4	High
Small	1.6851	1.5126	1.6140	2.1141	1.6395	10.1082	8.7661	9.2969	8.2701	9.1962
2	2.0772	1.7192	2.1014	1.7472	1.1454	9.0066	7.9045	9.0331	7.7572	8.8953
3	1.1125	1.3214	1.4360	1.0629	0.7175	7.4009	7.6881	7.7626	7.8410	9.9135
4	0.8620	1.0621	1.1183	1.0570	1.0488	7.3286	7.1012	6.5647	7.2185	8.2037
Big	0.2964	0.8184	0.4306	0.7031	0.4818	7.5539	7.3562	7.9723	7.8180	8.4644

Panel (c): Portfolios formed by size/profitability

	Mean					Standard deviation				
	Low	2	3	4	High	Low	2	3	4	High
Small	1.5424	1.9536	1.3257	1.8692	1.9659	10.7182	8.8083	8.3039	7.7938	10.1249
2	1.9084	1.2445	1.4908	1.6217	2.4318	9.7890	7.6491	7.4476	8.0016	8.9191
3	0.7834	1.1481	1.4690	1.1965	1.1571	8.5374	7.7659	7.4743	7.2863	8.8785
4	0.8954	0.7066	0.9109	1.2370	1.3156	7.0042	7.9307	6.4517	6.6165	7.8578
Big	0.3354	0.4861	0.3406	0.9200	0.5517	8.9496	7.0948	7.0585	7.9690	7.4829

Panel (d): Portfolios formed by size/AdjIlliq

	Mean					Standard deviation				
	Low	2	3	4	High	Low	2	3	4	High
Small	1.8617	1.6839	1.1982	1.9733	1.6145	10.1438	9.5912	8.1222	9.0275	7.7222
2	2.0591	2.0264	1.7457	1.6536	1.3425	9.7461	7.8073	7.4617	8.2574	8.9762
3	1.8324	0.7488	0.9997	1.3803	0.7841	7.3748	8.4058	7.4241	8.1945	8.3636
4	0.9757	1.0771	1.1184	1.2900	0.8324	6.5359	6.7242	6.0086	7.1153	8.9309
Big	0.8884	0.8292	0.5209	0.4254	0.5555	7.2188	6.3329	7.3091	7.1744	8.0973

Panel (e): Portfolios formed by size/ST

	Mean					Standard deviation				
	Low	2	3	4	High	Low	2	3	4	High
Small	2.2521	1.8876	1.6171	1.3842	1.1779	8.2807	8.1419	7.8809	9.3074	10.8519
2	1.7217	1.6720	1.7583	2.0482	1.5996	6.7725	6.5772	7.9984	9.6806	11.4090
3	1.5355	0.9960	1.5552	0.7503	0.9015	5.8401	6.3822	8.0621	8.8924	10.3852
4	1.3734	0.8736	0.9609	0.9047	1.0302	5.2795	5.9170	6.5813	7.5986	10.1066
Big	0.6254	0.8797	0.5434	0.3908	0.6408	7.1373	7.0743	6.9325	7.6391	9.7339

In each of the panels, the rows refer to the quintile by size and the columns refer to the quintile by the second variable considered in the formation of portfolios.

Finally, higher standard deviations for the excess returns of the *Small* portfolios were observed between the different LHS portfolio groups, compared to those in the *Big* portfolios. This shows that small-cap stocks have greater variability in returns; that is, they are riskier assets. As for the other variables used to construct the portfolios, the only case that revealed a pattern for the standard deviation was the ranking by Standardized Turnover – Panel (e), in which the *High* portfolios always showed greater standard deviations.

4.2. Factors

The next step in asset pricing is the construction of factors that constitute the independent variables of the models. Thus, it is also important to analyze the characteristics of the calculated factors. With this objective in mind, we present in Table 4 some statistics in order to analysis the factors excess returns from July 2000 to June 2018. Panel (a) presents the statistics of the factors of the Fama-French (1993) and Carhart (1997) models. The SMB factor showed the highest average return among all factors (0.8783%), while the IML factor, measured from the Standardized Turnover (IML_{ST}), showed the lowest average return (0.2247%). The average returns were only statistically significant for the SMB and $IML_{AdjIlliq}$ factors (at the 5% significance level).

Panel (b) of Table 4 presents the summary statistics for the factors of the Fama-French (2015) models. Again, the average returns for the SMB factor were the highest among all factors (0.9076%) and the average returns for the CMA factor were the lowest (0.1586%). Similar to data in Panel (a), only the average returns of the SMB and $IML_{AdjIlliq}$ factors were statistically significant (at the 5% significance level).

The summary statistics for the factors are similar to those of other studies that used data from emerging markets, such as Cakici, Fabozzi, and Tan (2013) and Leite et al. (2018), who, in general, observe a higher average for factor monthly returns in relation to the values observed in studies with a sample composed exclusively of developed markets. The average returns of the factors representing liquidity risk were both positive. Therefore, there is evidence that portfolios with less liquid stocks yield higher returns than portfolios with more liquid stocks. The results found for the liquidity risk factors are similar to those of Lam and Tam (2011) for the factors constructed from the Standardized Turnover and the Amihud (2002) Illiquidity measure. In the study by Lam and Tam (2011), only the average of the factor based on the Amihud (2002) Illiquidity proved to be significant; the average return of the factor built from the ST proxy was not significant.

Table 5 shows the correlations between factors. Panel (a) shows the correlations between the Fama and French (1993) and Carhart (1997) factors, and the liquidity factors. The HML and Market (R_m-R_f) factors showed the strongest positive correlation (0.4043), and the Market and IML_{ST} factors showed the strongest negative correlation (–0.7042).

Table 4. Factors excess returns analysis.

Panel (a): Summary statistics of the excess returns of the Fama-French three-factor model (1993), the momentum factor of the Carhart model (1997) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

	Rm-Rf	SMB	HML	WML	$IML_{AdjIlliq}$	IML_{ST}
Mean	0.7750	0.8783	0.4460	0.5629	0.6731	0.2247
Standard deviation	6.9756	2.7187	3.9544	4.8929	3.4277	4.5317
Skewness	−0.9295	0.0852	−0.0215	−0.3034	1.1329	−0.0001
Kurtosis	3.2129	−0.0090	1.1484	1.6118	5.6402	1.2753
t-statistic	1.6328	4.7480	1.6577	1.6909	2.8859	0.7287
P-value	0.1040	0.0000	0.0988	0.0923	0.0043	0.4670

Panel (b): Summary statistics of the excess returns of the Fama-French five-factor model (2015), the momentum factor of the Carhart model (1997) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

	Rm-Rf	SMB	HML	CMA	RMW	$IML_{AdjIlliq}$	IML_{ST}
Mean	0.7750	0.9076	0.4460	0.1586	0.2749	0.6731	0.2247
Standard deviation	6.9756	2.7721	3.9544	4.1325	3.3548	3.4277	4.5317
Skewness	−0.9295	0.1468	−0.0215	0.0445	−0.9242	1.1329	−0.0001
Kurtosis	3.2129	−0.0359	1.1484	1.3586	3.3941	5.6402	1.2753
t-statistic	1.6328	4.8117	1.6577	0.5640	1.2044	2.8859	0.7287
P-value	0.1040	0.0000	0.0988	0.5733	0.2297	0.0043	0.4670

Panel (a) presents summary statistics of the excess returns for the Fama-French three-factor model (1993), the momentum factor of the Carhart model (1997) and the $IML_{AdjIlliq}$ and IML_{ST} factors. Given that the model proposed by Carhart (1997) only incorporated the WML factor into the Fama-French model (1993), the separate analysis of the factors in this model was not necessary. Panel (b) presents summary statistics of the excess returns of the Fama-French five-factor model (2015) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

Table 5. Correlations between the factors.

Panel (a): Correlation between the Fama-French three-factor model (1993), the momentum factor of the Carhart model (1997) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

	Rm-Rf	SMB	HML	WML	$IML_{AdjIlliq}$	IML_{ST}
Rm-Rf	1					
SMB	−0.1525	1				
HML	0.4043	0.0306	1			
WML	−0.1411	−0.1187	−0.3166	1		
$IML_{AdjIlliq}$	−0.2831	0.3006	−0.3169	0.1582	1	
IML_{ST}	−0.7042	0.0206	−0.5133	0.2905	0.3613	1

Panel (b): Correlation between the Fama-French five factors (2015) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

	Rm-Rf	SMB	HML	RMW	CMA	$IML_{AdjIlliq}$	IML_{ST}
Rm-Rf	1						
SMB	−0.1111	1					
HML	0.4043	0.0359	1				
RMW	−0.1346	−0.2194	−0.2699	1			
CMA	−0.2108	0.0428	0.1773	−0.2243	1		
$IML_{AdjIlliq}$	−0.2831	0.3220	−0.3169	−0.1510	0.0768	1	
IML_{ST}	−0.7042	−0.0305	−0.5133	0.2983	0.0682	0.3613	1

Panel (a) presents the correlations between the Fama-French three factors (1993), the momentum factor of the Carhart model (1997) and the $IML_{AdjIlliq}$ and IML_{ST} factors. Given that the model proposed by Carhart (1997) only incorporated the WML factor into the Fama-French model (1993), the separate analysis of the factors in this model was not necessary. Panel (b) presents the correlations between the Fama-French five-factor model (2015) and the $IML_{AdjIlliq}$ and IML_{ST} factors.

The correlation estimates between the Fama-French (2015) factors and the liquidity factors – show in Panel (b) – are similar to those in Panel (a). We highlight the correlation between profitability and investment and liquidity factors. The profitability factor – RMW – showed a negative correlation with $IML_{AdjIlliq}$ (−0.1510) and a positive correlation with IML_{ST} (0.2983). The investment factor – CMA – showed a positive and weak correlation with both liquidity factors, that is, 0.0768 with the $IML_{AdjIlliq}$ factor and 0.0682 with the IML_{ST} factor.

Regarding the liquidity risk factors, we highlight the weak correlation between $IML_{AdjIlliq}$ and IML_{ST} (0.3613), which indicates that they capture different dimensions of liquidity. Furthermore, in absolute values, the HML factor showed the highest correlation with $IML_{AdjIlliq}$ (−0.3169), and that Market is the factor with the highest correlation with IML_{ST} (−0.7042).

5. Asset Pricing Model Evaluations

This section presents the results of the statistics used to evaluate the performance of the pricing models estimated. Table 6 shows the results of the statistics used to evaluate the asset pricing models – the Fama-French five-factor model (2015), and the Fama and French (1993) and Carhart (1997) three and four-factor models, respectively. Each Panel in this Table presents the results found for a different group of LHS portfolios.

The GRS statistic of Gibbons, Ross, and Shanken (1989) is performed under the null hypothesis that all intercepts of the tested pricing model are jointly statistically equal to zero; that is, that the model fully explains the excess return of the portfolios. From the GRS test, the best asset pricing model is the one with the lowest test statistic and the highest *p*-value.

The results in Table 6 show that the models incorporating the IML_{ST} factor showed greater explanatory power compared to the models that considered the $IML_{AdjIlliq}$ factor. In general, the highest *p*-value in the GRS test referred to the model incorporating the IML_{ST} factor for liquidity risk. The highest *p*-value (0.5044) only referred to the model with the $IML_{AdjIlliq}$ in the set of portfolios formed according to size and profitability, as shown in Panel (c).

Among the different groups of LHS portfolios in Table 6, it appears that the models with the worst performance (high GRS statistics and small p-values) in explaining the returns are those in Panel (e); that is, the portfolios formed according to Size and Standardized Turnover. In all models estimated for this set of portfolios, the null hypothesis of the GRS test was rejected (at the 5% significance level) and the highest p-value was observed in the five-factor model incorporating the IML_{ST} factor (0.0243). For the other groups, there were always models in which the GRS test null hypothesis could not be rejected, thus indicating that they were able to fully explain the returns.

Table 6. Models' evaluation statistics.

Panel (a): Portfolios formed according to size/book-to-market ratio						
	GRS	p-value	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	1.9574	0.0063	0.2991	0.4161	0.6214	0.7691
Three-factor + IML_{ST}	1.5236	0.0611	0.2508	0.3731	0.5212	0.7675
Four-factor + $IML_{AdjIlliq}$	1.8452	0.0118	0.2817	0.4071	0.5854	0.7719
Four-factor + IML_{ST}	1.5035	0.0673	0.2477	0.3741	0.5147	0.7701
Five-factor + $IML_{AdjIlliq}$	1.6655	0.0304	0.2783	0.3894	0.5782	0.7740
Five-factor + IML_{ST}	1.3756	0.1202	0.2453	0.3513	0.5098	0.7733
Panel (b): Portfolios formed according to size/investment						
	GRS	p-value	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.9960	0.4746	0.1929	0.2738	0.4483	0.7282
Three-factor + IML_{ST}	0.9538	0.5314	0.1911	0.2766	0.4441	0.7299
Four-factor + $IML_{AdjIlliq}$	1.3451	0.1368	0.2416	0.3297	0.5614	0.7333
Four-factor + IML_{ST}	1.2938	0.1693	0.2323	0.3354	0.5399	0.7345
Five-factor + $IML_{AdjIlliq}$	0.9017	0.6030	0.2103	0.2812	0.4888	0.7594
Five-factor + IML_{ST}	0.8862	0.6242	0.2123	0.2649	0.4935	0.7611
Panel (c): Portfolios formed according to size/profitability						
	GRS	p-value	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	1.3677	0.1241	0.2566	0.3513	0.5990	0.7464
Three-factor + IML_{ST}	1.2626	0.1918	0.2522	0.3277	0.5885	0.7483
Four-factor + $IML_{AdjIlliq}$	1.3094	0.1589	0.2347	0.3421	0.5477	0.7499
Four-factor + IML_{ST}	1.2437	0.2066	0.2286	0.3238	0.5336	0.7516
Five-factor + $IML_{AdjIlliq}$	0.9737	0.5044	0.2122	0.2485	0.4952	0.7620
Five-factor + IML_{ST}	1.1079	0.3373	0.2458	0.2842	0.5738	0.7632
Panel (d): Portfolios formed according to size/AdjIlliq						
	GRS	p-value	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	1.3949	0.1102	0.2305	0.2813	0.5353	0.7627
Three-factor + IML_{ST}	1.2924	0.1702	0.2187	0.2829	0.5079	0.7556
Four-factor + $IML_{AdjIlliq}$	1.2699	0.1864	0.2090	0.2579	0.4852	0.7647
Four-factor + IML_{ST}	1.1981	0.2456	0.2123	0.2614	0.4931	0.7577
Five-factor + $IML_{AdjIlliq}$	1.2616	0.1927	0.2298	0.2866	0.5335	0.7666
Five-factor + IML_{ST}	1.2462	0.2047	0.2267	0.2839	0.5264	0.7601
Panel (e): Portfolios formed according to size/ST						
	GRS	p-value	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	2.0283	0.0042	0.3586	0.4741	0.8171	0.7457
Three-factor + IML_{ST}	1.7369	0.0209	0.2650	0.3467	0.6037	0.7582
Four-factor + $IML_{AdjIlliq}$	1.9050	0.0085	0.3388	0.4571	0.7718	0.7485
Four-factor + IML_{ST}	1.7121	0.0239	0.2478	0.3494	0.5646	0.7613
Five-factor + $IML_{AdjIlliq}$	1.7939	0.0156	0.3440	0.4683	0.7838	0.7506
Five-factor + IML_{ST}	1.7093	0.0243	0.2882	0.3703	0.6566	0.7632

The GRS statistic and p-value columns refer to the results found through the Gibbons, Ross and Shanken test. Columns $A|\alpha_i|$ and R^2 refer to the average of the absolute values of the intercepts and determination coefficients of the models, respectively. The column $s(\alpha)$ shows the standard deviation of the intercept values of the models. Finally, the column $A|\alpha_i|/ A|r_i|$ shows the average absolute value of the intercepts over the average absolute value of the average return of portfolio i, minus the average returns of all portfolios formed from the same variables considered in the construction of portfolio i.

Furthermore, the five-factor model incorporating the IML_{ST} factor was the one that had the best performance most times. The *p*-values of this test were higher in the groups formed (in the second ranking) according to: (i) book-to-market ratio (p = .12029); (ii) investment (p = .6242); and (iii) Standardized Turnover (p = .0243). For portfolios formed according to size and profitability, the highest *p*-value found was that of the five-factor model incorporating the $IML_{AdjIlliq}$ factor (0.5044). For portfolios formed according to Adjusted Illiquidity, the four-factor model with the addition of the IML_{ST} factor showed the highest *p*-value (p = .2456).

Table 6 also shows the average absolute values of intercepts produced by a given model for a set of LHS portfolios – $A|a_i|$. The results found for this statistic represent yet another evidence favorable to the IML_{ST} factor. In evaluating the addition of the liquidity factor to the models, the model evaluated with the $IML_{AdjIlliq}$ factor presented a lower average absolute value of intercept than the model with the IML_{ST} factor in only three cases: for the five-factor model in the size/investment portfolios; for the five-factor model in the size/profitability portfolios; and for the four-factor model in the size/AdjIlliq portfolios.

Another statistic in Table 6 is the dispersion of the models intercepts in relation to the expected return for a group of LHS portfolios – $A|a_i|/A|r_i|$. The numerator for this measure is the average absolute value of the intercepts – $A|a_i|$. The denominator – $A|r_i|$ – is the average absolute deviation of the returns on each LHS portfolio *i* from the average returns on all LHS portfolios formed from the same variables considered in the construction of the LHS portfolio *i*. As described by Fama and French (2015), this statistic shows how much of the expected returns of the LHS portfolios remain unexplained by the competing factor models.

Lower $A|a_i|/A|r_i|$ values indicate that the model left a smaller portion of the average expected returns unexplained. The results for this statistic were not unanimous in pointing out a model or liquidity factor that stood out as the best among the LHS portfolios. In each Panel in Table 6, a different model showed the lowest value for $A|a_i|/A|r_i|$.

Lastly, the average R^2 of the models of the LHS portfolio groups represents a favorable result to Fama-French models (2015). These results demonstrate an alternation between liquidity factors in the models with the highest average R^2 for each group of portfolios. For the portfolios in Panels (a) and (d), the best model was the Fama-French five-factor model (2015) incorporating the $IML_{AdjIlliq}$ factor (R^2 average values of 0.7740 and 0.7666, respectively). For the other groups of portfolios, the best model was the five-factor model incorporating the IML_{ST} factor [R^2 = 0.7611 in Panel (b); R^2 = 0.7632 in Panels (c) and (e)].

In summary, the results in Table 6 show favorable evidence for the inclusion of a liquidity factor in asset pricing models. In the studied sample, the IML_{ST} factor led to a greater impact on the explanatory power of the models, i.e., its inclusion in the models increased the proportion of variation in the excess return of LHS portfolios explained by the models. This corroborates existing empirical evidence of a liquidity premium, such as Lam and Tam (2011) and Machado and Medeiros (2011), which also found favorable evidence to the addition of a liquidity factor to asset pricing models. There is also favorable evidence to the models based on the Fama-French five-factor model, which in most cases showed greater explanatory power regarding stock returns.

5.1. Robustness Analysis

To verify the robustness of the results, the models were estimated again, disregarding the data corresponding to January and splitting the sample period into two equal subperiods: (i) from July 2000 to June 2009; and (ii) from July 2009 to June 2018. The GRS test results for the models estimated for this purpose are presented in Appendix A. The models were also estimated using the Racicot (2015) GMM-IV_d and the results for the model's evaluation statistics are presented in Appendix B.

In general, the estimates obtained through the models without January data and with the two subsamples showed no major changes in the results in terms of the performance of the models. Once again, the IML_{ST} factor proved to be superior, given that, as a rule, the models with the highest *p*-value for the GRS test were those incorporating this liquidity factor. In most cases, the model with the highest p-value for the GRS test was the one based on the Fama-French five-factor model. However, among the models estimated from data corresponding to the period between July 2009 and June 2018, the model with the highest p-value was, in most cases, the one based on the Carhart four-factor model.

As for the January Effect, the results of the GRS test for the models presented in Appendix A were similar to those observed when the models were estimated with the total sample. This evidence contradicts the existence of the January Effect, in line with the results of Liu (2006), Machado and Medeiros (2011) and Lam and Tam (2011).

The greatest impact on the p-value of the GRS test was observed in the models estimated from the sample period divided into two subperiods. For the models estimated for the first sample period from July 2000 to June 2009 – Pre-2009 column in Appendix A –, the p-value of the GRS test plunged, thus indicating a tendency to reject its null hypothesis. In turn, the GRS test for the models estimated for the post-2009 sample pointed to an increase in p-values. These higher values indicate better model's performances in explaining the LHS portfolio returns on the second sample period.

This empirical evidence indicates that liquidity effect varies over time as pointed out by Amihud (2002, 2019), and in this case liquidity becomes a more significant factor over the sample period. The critical and uncertain scenario that affected the Latin American emerging markets surveyed herein, especially in the second half of the sample period, may have contributed to this result. As highlighted by Rösch and Kaserer (2013), in scenarios of greater uncertainty, investors tend to seek protection in assets with greater liquidity, an effect that is referred to as "flight-to-liquidity." Another potential explanation for the liquidity impact on stock returns could be provided by varying degrees of integration of emerging stock markets with the world economy as argued by Jun, Marathe, and Shawky (2003). In other words, the trend observed in the subsample evaluation may be reflecting a greater level of integration of Latin American emerging capital markets with the global market.

When analyzed the average R^2 from the GMM-IV_d models, presented in the Appendix B, again the results were favorable to the five-factor model with the IML_{ST} liquidity factor. But the statistics $A|\alpha_i|$ and $A|\alpha_i|/A|r_i|$ from the GMM-IV_d were ambiguous for (i) the five and four-factor models; and (ii) the models with IML_{ST} or $IML_{AdjIlliq}$ liquidity factors. Thus, we could not identify which of these models left unexplained a smaller portion of the average expected returns.

6. Conclusion

The assessment of the role of liquidity risk in asset pricing is frequent in financial literature. However, studies on this relationship in the context of emerging markets, especially in Latin American ones, are scarce. Given this gap in the literature, this study aimed to test the performance of factor pricing models with the addition of a stock liquidity factor in the context of Latin America emerging markets. To estimate the models, we considered the portfolio returns of stocks listed on the markets of Brazil, Chile, Colombia, Mexico and Peru, between July 2000 and June 2018, formed according to the procedures of Fama and French (2015).

The results of the GRS test demonstrated that the addition of a liquidity factor to the Frama-French five-factor model (2015), as well the three and four-factor models by Fama and French (1993) and Carhart (1997), were able to increase the model performance compared to their original empirical structures. In general, the models with the liquidity factor constructed following Standardized Turnover (IML_{ST}) showed greater explanatory power. Likewise, this evidence favors the adoption of this factor to the detriment of the one constructed following Adjusted Illiquidity ($IML_{AdjIlliq}$).

As for the performance of the models among the LHS portfolio groups, the least explanatory power was observed in the portfolios formed according to size and Standardized Turnover. This result indicates that other relevant factors could aid to explain the excess returns of such portfolios. In the other LHS portfolio groups, it was always possible to identify at least one model for what the null hypothesis of the GRS test was not rejected, thus indicating that the factors considered were able to explain the expected returns.

When analyzing the results of the GRS test, the five-factor model incorporating the IML_{ST} factor proved to be more robust for portfolios formed according to book-to-market ratio, investment and Standardized Turnover criteria in the second ranking. For portfolios based on size and profitability, the most robust model was the Fama-French five-factor model (2015) incorporating the $IML_{AdjIlliq}$ factor. For portfolios formed according to size and Adjusted Illiquidity, the model with the highest *p*-value in the GRS test was the Carhart model (1997) incorporating the IML_{ST} factor.

Regarding the robustness of the results, the January Effect has not been detected, which corroborates the empirical evidence of Liu (2006), Machado and Medeiros (2011) and Lam and Tam (2011). When the models were estimated again with the sample period split into two subperiods, the results pointed to a more significant liquidity effect in the second half of the sample period (between July 2009 and June 2018). The economic crises that impacted Latin American emerging economies during the sample period may have contributed to this result, given that the flight-to-liquidity effect tends to be observed in scenarios of greater uncertainty. In this case, investors tend to seek safer and more liquid markets. Moreover, the greater importance of liquidity in a given market may be associated with its level of integration into the global market. Therefore, this may also indicate that the surveyed Latin American emerging markets have undergone a process of integration into the global economy over the study timeframe.

In face of the possible endogeneity problem in asset pricing models with liquidity, we also estimated our models by the Racicot's GMM-IV_d. Altogether, the robust instruments proposed by Racicot (2015) were strong. But the results where ambiguous in terms of indicating which model has the best performance, the five or four-factor models, and the model with the IML_{ST} or the $IML_{AdjIlliq}$ liquidity factor.

The empirical evidence in this article shows that liquidity is a priced risk factor in Latin American emerging markets, and points to the pertinence of a six-factor pricing model. These results contribute to a better understanding of the relevant factors in emerging markets and their impact on asset pricing, especially for the effect of market liquidity on the expected return by investors. As a result, financial market players have better conditions for making decisions both at the time of portfolios formation, and when assessing its performance in relation to the different sources of systemic risk they are exposed.

This article adds two important contributions to the literature on asset pricing. First, we highlight the incorporation of a liquidity factor into the Fama-French five-factor model. Second, this study distinguishes itself by adopting the measures of Liu (2006) and Kang and Zhang (2014) for the construction of liquidity risk factors, which increase the accuracy of liquidity measurement in emerging stock markets.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendices

Appendix A. Robustness Analysis Results

Panel (a): Portfolios formed according to size/book-to-market ratio	Without January Data		Pre-2009		Post-2009	
	GRS	p-value	GRS	p-value	GRS	p-value
Three-factor + $IML_{AdjIlliq}$	1.7646	0.0190	1.9918	0.0112	1.9397	0.0141
Three-factor + IML_{ST}	1.5132	0.0659	1.7776	0.0287	1.4483	0.1101
Four-factor + $IML_{AdjIlliq}$	1.6341	0.0369	1.9422	0.0142	1.6735	0.0449
Four-factor + IML_{ST}	1.4425	0.0911	1.7548	0.0319	1.3047	0.1873
Five-factor + $IML_{AdjIlliq}$	1.5392	0.0585	1.7863	0.0282	1.5856	0.0648
Five-factor + IML_{ST}	1.3598	0.1307	1.6258	0.0551	1.3799	0.1434
Panel (b): Portfolios formed according to size/investment	Without January Data		Pre-2009		Post-2009	
	GRS	p-value	GRS	p-value	GRS	p-value
Three-factor + $IML_{AdjIlliq}$	0.9253	0.5705	1.1323	0.3296	1.0546	0.4129
Three-factor + IML_{ST}	0.9530	0.5327	1.1001	0.3628	0.7063	0.8355
Four-factor + $IML_{AdjIlliq}$	1.2884	0.1752	1.1440	0.3184	0.9147	0.5851
Four-factor + IML_{ST}	1.2675	0.1903	1.1074	0.3555	0.7273	0.8136
Five-factor + $IML_{AdjIlliq}$	0.8666	0.6507	1.0532	0.4151	0.6591	0.8793
Five-factor + IML_{ST}	0.8964	0.6100	1.0421	0.4278	0.6079	0.9190
Panel (c): Portfolios formed according to size/profitability	Without January Data		Pre-2009		Post-2009	
	GRS	p-value	GRS	p-value	GRS	p-value
Three-factor + $IML_{AdjIlliq}$	1.2942	0.1710	1.8392	0.0220	1.6005	0.0602
Three-factor + IML_{ST}	1.2435	0.2087	1.9014	0.0168	1.0519	0.4161
Four-factor + $IML_{AdjIlliq}$	1.2838	0.1784	1.8011	0.0262	1.3423	0.1639
Four-factor + IML_{ST}	1.2687	0.1894	1.9011	0.0170	0.9446	0.5468
Five-factor + $IML_{AdjIlliq}$	0.8908	0.6177	1.5863	0.0646	1.2520	0.2252
Five-factor + IML_{ST}	1.0163	0.4491	1.6940	0.0416	1.0531	0.4152
Panel (d): Portfolios formed according to size/AdjIlliq	Without January Data		Pre-2009		Post-2009	
	GRS	p-value	GRS	p-value	GRS	p-value
Three-factor + $IML_{AdjIlliq}$	1.4868	0.0744	1.4998	0.0901	1.6142	0.0569
Three-factor + IML_{ST}	1.4359	0.0937	1.4064	0.1290	1.5106	0.0864
Four-factor + $IML_{AdjIlliq}$	1.3434	0.1399	1.4647	0.1038	1.3161	0.1800
Four-factor + IML_{ST}	1.3114	0.1596	1.4117	0.1270	1.2399	0.2341
Five-factor + $IML_{AdjIlliq}$	1.4066	0.1069	1.3364	0.1679	1.4409	0.1142
Five-factor + IML_{ST}	1.3874	0.1162	1.2472	0.2289	1.5843	0.0652
Panel (e): Portfolios formed according to size/ST	Without January Data		Pre-2009		Post-2009	
	GRS	p-value	GRS	p-value	GRS	p-value
Three-factor + $IML_{AdjIlliq}$	1.8215	0.0141	1.1459	0.3162	1.9967	0.0110
Three-factor + IML_{ST}	1.6209	0.0393	1.0125	0.4624	1.5236	0.0820
Four-factor + $IML_{AdjIlliq}$	1.6668	0.0314	1.1105	0.3522	1.7268	0.0359
Four-factor + IML_{ST}	1.5499	0.0555	0.9956	0.4831	1.3772	0.1443
Five-factor + $IML_{AdjIlliq}$	1.6640	0.0319	1.0156	0.4591	1.5910	0.0634
Five-factor + IML_{ST}	1.6556	0.0332	0.9198	0.5785	1.4643	0.1045

The Without January Data column refer to the results of the January Effect Test and the Pre-2009 and Post 2009 columns refer to the results of the Two-Subperiods Sample Estimate. For each estimate, the GRS and *p*-value columns refer to the results found through the Gibbons, Ross and Shanken test.

Appendix B. GMM-IV_d Evaluation Statistics

Panel (a): Portfolios formed according to size/book-to-market ratio				
	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.3579	0.4936	0.7438	0.7300
Three-factor + IML_{ST}	0.3001	0.4595	0.6236	0.7430
Four-factor + $IML_{AdjIlliq}$	0.2807	0.4068	0.5833	0.7664
Four-factor + IML_{ST}	0.2397	0.3699	0.4980	0.7644
Five-factor + $IML_{AdjIlliq}$	0.2782	0.3894	0.5781	0.7674
Five-factor + IML_{ST}	0.2420	0.3488	0.5028	0.7666
Panel (b): Portfolios formed according to size/investment				
	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.2625	0.3261	0.6100	0.6982
Three-factor + IML_{ST}	0.2415	0.3402	0.5612	0.6965
Four-factor + $IML_{AdjIlliq}$	0.2423	0.3299	0.5632	0.7269
Four-factor + IML_{ST}	0.2294	0.3349	0.5331	0.7278
Five-factor + $IML_{AdjIlliq}$	0.2066	0.2792	0.4801	0.7525
Five-factor + IML_{ST}	0.2014	0.2526	0.4681	0.7537
Panel (c): Portfolios formed according to size/profitability				
	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.3214	0.4952	0.7500	0.7071
Three-factor + IML_{ST}	0.3567	0.4284	0.8324	0.7196
Four-factor + $IML_{AdjIlliq}$	0.2359	0.3447	0.5506	0.7439
Four-factor + IML_{ST}	0.2225	0.3169	0.5193	0.7455
Five-factor + $IML_{AdjIlliq}$	0.2121	0.2507	0.4950	0.7551
Five-factor + IML_{ST}	0.2480	0.2890	0.5788	0.7561
Panel (d): Portfolios formed according to size/AdjIlliq				
	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.3123	0.3538	0.7252	0.7270
Three-factor + IML_{ST}	0.3082	0.3906	0.7156	0.7362
Four-factor + $IML_{AdjIlliq}$	0.2116	0.2605	0.4913	0.7590
Four-factor + IML_{ST}	0.2195	0.2670	0.5097	0.7517
Five-factor + $IML_{AdjIlliq}$	0.2323	0.2884	0.5395	0.7598
Five-factor + IML_{ST}	0.2313	0.2889	0.5371	0.7529
Panel (e): Portfolios formed according to size/ST				
	$A\|\alpha_i\|$	$s(\alpha)$	$A\|\alpha_i\|/A\|r_i\|$	R^2
Three-factor + $IML_{AdjIlliq}$	0.4697	0.5662	1.0702	0.7047
Three-factor + IML_{ST}	0.3619	0.4901	0.8246	0.7324
Four-factor + $IML_{AdjIlliq}$	0.3406	0.4593	0.7761	0.7425
Four-factor + IML_{ST}	0.2504	0.3599	0.5704	0.7551
Five-factor + $IML_{AdjIlliq}$	0.3445	0.4693	0.7848	0.7433
Five-factor + IML_{ST}	0.2982	0.3894	0.6794	0.7559

Columns $A|\alpha_i|$ and R^2 refer to the average of the absolute values of the intercepts and determination coefficients of the models, respectively. The column $s(\alpha)$ shows the standard deviation of the intercept values of the models. Finally, the column $A|\alpha_i|/ A|r_i|$ shows the average absolute value of the intercepts over the average absolute value of the average return of portfolio *i*, minus the average returns of all portfolios formed from the same variables considered in the construction of portfolio *i*.